More Integral Practice

MA2160 Spring 2007

February 13, 2007

1. $\int x\sqrt{x^2+1}\,dx$

   Solution. Let $w = x^2+1$. Then $dw = 2xdx$.

$$\int x\sqrt{x^2+1}\,dx = \frac{1}{2}\int w^{\frac{1}{2}}\,dw = \frac{w^{\frac{3}{2}}}{3} + c = \frac{(x^2+1)^{\frac{3}{2}}}{3} + c$$

□

2. $\int x\sqrt{5-x}\,dx$

   Solution. Let $w = 5-x$. Then $dw = -dx$.

$$\int x\sqrt{5-x}\,dx = -\int (5-w)\sqrt{w}\,dw = -\int 5w^{\frac{1}{2}} - w^{\frac{3}{2}}\,dw = \frac{-10w^{\frac{3}{2}}}{3} + \frac{2w^{\frac{5}{2}}}{5} + c$$

$$= \frac{-10(5-x)^{\frac{3}{2}}}{3} + \frac{2(5-x)^{\frac{5}{2}}}{5} + c$$

□

3. $\int \sec^2(7x)\tan^4(7x)\,dx$

   Solution. Let $w = \tan(7x)$. Then $dw = 7\sec^2(7x)dx$.

$$\int \sec^2(7x)\tan^4(7x)\,dx = \frac{1}{7}\int w^2\,dw = \frac{w^3}{21} + c = \frac{\tan^3(7x)}{21} + c$$

□

4. $\int_0^1 \frac{4x+12}{x^2-6x+9}\,dx$

   Solution. By partial fraction theory,

$$\frac{4x+12}{(x-3)^2} = \frac{A}{x-3} + \frac{B}{(x-3)^2}.$$

$$4x+12 = A(x-3)+B$$

$$4x + 12 = Ax - 3A + B$$

Then $A = 4$ and $B = 24$.

$$\int_0^1 \frac{4x+12}{x^2-6x+9}\,dx = \int_0^1 \frac{4}{x-3}\,dx + \int_0^1 \frac{24}{(x-3)^2}\,dx = 4\ln|x-3|\Big|_0^1 - \frac{24}{x-3}\Big|_0^1 = 4\ln 2 - 4\ln 3 + 4$$

□

5. $\int_2^4 \frac{5}{x\ln x}\,dx$

Solution. Let $w = \ln x$. Then $dw = \frac{1}{x}dx$.

$$\int_2^4 \frac{5}{x\ln x}\,dx = 5\int_{\ln 2}^{\ln 4} \frac{dw}{w} = 5\ln|w|\Big|_{\ln 2}^{\ln 4} = 5(\ln(\ln 4) - \ln(\ln 2))$$

□

6. $\int_0^1 \frac{5}{x^2+5x-14}\,dx$

Solution. By partial fraction theory,

$$\frac{5}{(x-2)(x+7)} = \frac{A}{x-2} + \frac{B}{x+7}$$

$$5 = A(x+7) + B(x-2)$$

$$0x + 5 = (A+B)x + (7A - 2B)$$

Then $A = \frac{5}{9}$ and $B = -\frac{5}{9}$.

$$\int_0^1 \frac{5}{x^2+5x-14}\,dx = \frac{5}{9}\int_0^1 \frac{dx}{x-2} - \frac{5}{9}\int_0^1 \frac{dx}{x+7} = \frac{5}{9}\ln|x-2|\Big|_0^1 - \frac{5}{9}\ln|x+7|\Big|_0^1$$

$$= -\frac{5}{9}\ln 2 - \frac{5}{9}\ln 8 + \frac{5}{9}\ln 7$$

□

7. $\int x^3 e^{x^2}\,dx$

Solution. Let $u = x^2$ and $v' = xe^{x^2}dx$. Then $u' = 2xdx$ and $v = \frac{e^{x^2}}{2}$.

$$\int x^3 e^{x^2}\,dx = \frac{x^2 e^{x^2}}{2} - \int xe^{x^2}\,dx = \frac{x^2 e^{x^2}}{2} - \frac{1}{2}e^{x^2} + c$$

□

8. $\int \frac{\sin(\ln x)}{x}\, dx$

Solution. Let $w = \ln x$. Then $dw = \frac{1}{x}dx$.

$$\int \frac{\sin(\ln x)}{x}\, dx = \int \sin(w)\, dw = -\cos w + c = -\cos(\ln x) + c$$

□

9. $\int \frac{1+\ln y}{y}\, dy$

Solution. Let $w = 1 + \ln y$. Then $dw = \frac{1}{y}dy$.

$$\int \frac{1+\ln y}{y}\, dy = \int w\, dw = \frac{w^2}{2} + c = \frac{(1+\ln y)^2}{2} + c$$

□

10. $\int e^{2x} \sin(3x)\, dx$

Solution. Let $u = \sin(3x)$ and $v' = e^{2x}dx$. Then $u' = 3\cos(3x)$ and $v = \frac{e^{2x}}{2}$.

$$\frac{\sin(3x)e^{2x}}{2} - \frac{3}{2}\int e^{2x}\cos(3x)\, dx$$

Let $u = \cos(3x)$ and $v' = e^{2x}dx$. Then $u' = -3\sin(3x)$ and $v = \frac{e^{2x}}{2}$.

$$\int e^{2x}\sin(3x)\, dx = \frac{\sin(3x)e^{2x}}{2} - \frac{3}{2}\left[\frac{\cos(3x)e^{2x}}{2} + \frac{3}{2}\int e^{2x}\sin(3x)\, dx\right]$$

$$= \frac{\sin(3x)e^{2x}}{2} - \frac{3\cos(3x)e^{2x}}{4} - \frac{9}{4}\int e^{2x}\sin(3x)\, dx$$

Solving for the integral,

$$\frac{13}{4}\int e^{2x}\sin(3x)\, dx = \frac{\sin(3x)e^{2x}}{2} - \frac{3\cos(3x)e^{2x}}{4}$$

$$\int e^{2x}\sin(3x)\, dx = \frac{2\sin(3x)e^{2x}}{13} - \frac{3\cos(3x)e^{2x}}{13} + c$$

□

11. $\int x\cos(7x)\, dx$

Solution. Let $u = x$ and $v' = \cos(7x)dx$. Then $u' = dx$ and $v = \frac{1}{7}\sin(7x)$.

$$\int x\cos(7x)\, dx = \frac{x\sin(7x)}{7} - \frac{1}{7}\int \sin(7x)\, dx = \frac{x\sin(7x)}{7} + \frac{\cos(7x)}{49} + c$$

□

12. $\int \frac{\sin(\pi\sqrt{t})}{\sqrt{t}}\ dt$

Solution. Let $w = \pi\sqrt{t}$. Then $dw = \frac{\pi}{2\sqrt{t}}dt$.

$$\int \frac{\sin(\pi\sqrt{t})}{\sqrt{t}}\ dx = \frac{2}{\pi}\int \sin(w)\ dw = -\frac{2}{\pi}\cos(w) + c = -\frac{2\cos(\pi\sqrt{t})}{\pi} + c$$

□

13. $\int \frac{6x^3+4x+3}{x}\ dx$

Solution.

$$\int \frac{6x^3+4x+3}{x}\ dx = \int \left(6x^2 + 4 + \frac{3}{x}\right)\ dx = 2x^3 + 4x + 3\ln|x| + c$$

□

14. $\int_1^2 \frac{x+4}{x^2+8x}\ dx$

Solution. By partial fraction theory,

$$\frac{x+4}{x(x+8)} = \frac{A}{x} + \frac{B}{x+8}$$
$$x + 4 = A(x+8) + Bx$$
$$x + 4 = (A+B)x + 8A$$

Then $A = \frac{1}{2}$ and $B = \frac{7}{8}$.

$$\int_1^2 \frac{x+4}{x^2+8x}\ dx = \frac{1}{2}\int_1^2 \frac{1}{x}\ dx + \frac{7}{8}\int_1^2 \frac{1}{x+8}\ dx = \frac{1}{2}\ln|x|\Big|_1^2 + \frac{7}{8}\ln|x+8|\Big|_1^2$$
$$= \frac{1}{2}\ln 2 + \frac{7}{8}(\ln 10 - \ln 9) + c$$

□

15. $\int t^2 e^t\ dt$

Solution. Let $u = t^2$ and $v' = e^t dt$. Then $u' = 2tdt$ and $v = e^t$.

$$\int t^2 e^t\ dt = t^2 e^t - 2\int te^t\ dt$$

Let $u = t$ and $v' = e^t dt$. Then $u' = dt$ and $v = e^t$.

$$\int t^2 e^t\ dt = t^2 e^t - 2\left[te^t - \int e^t\ dt\right] = t^2 e^t - 2te^t + 2e^t + c$$

□